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Every Student Deserves a Struggle: Planning for the Implementation and Differentiation of High Cognitive Tasks

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Every Student Deserves a Struggle: Planning for the Implementation & Differentiation of High Cognitive Tasks

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Every Student Deserves a Struggle: Planning for the Implementation & Differentiation of High Cognitive Tasks

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Abstract

The work of Mary Kay Stein and Margaret Smith is foundational to cognitive research in education. Most other works in the field branch out from their work in one way or another. One of their most influential contributions was their system for classifying tasks according to cognitive demand or the type of thinking the tasks requires students to engage in. Memorization tasks or tasks that involve procedural work without making deeper connections are classified as having low cognitive demand. On the other hand, tasks involving procedures with connections and “doing” mathematics tasks are viewed as having high cognitive demand. Research in the field has examined the nature of the different kinds of tasks and teachers’ ability to identify tasks by these categories. The goal was to promote a greater use of high cognitive tasks in the classroom. As more high cognitive tasks were being selected by teachers, it became apparent that selection did not guarantee student engagement at the intended level. Stein and Smith also worked extensively with the Mathematical Task Framework, which describes the transformation a task may take from curriculum to planning to implementation. This framework led to the ability for researchers to better understand when changes might occur and what factors affected the maintenance or decline of demand.

Two main challenges emerged from the existing research that will be addressed within this project. Who benefits from high cognitive demand tasks and how the implementation of tasks can be planned in a way that will maintain a high level of demand. The goal of this project is to design high cognitive demand tasks that can be differentiated to meet the needs of all students and provide suggestions during the planning phase for how tasks can be implemented to maintain high levels of cognitive demand.

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Introduction

Every student recalls a teacher that made a difference in their life. For me, it was my eighth-grade math teacher, Mr. O'Connell. He was not a traditional middle school teacher. In fact, teaching was his second career after serving in the Coast Guard. He was a brilliant man and had a way of pushing limits. He once told my class about a drill he conducted with his unit, and I still smile every time I envision the mayhem. He jumped overboard to challenge his crew in a simulation of a rescue mission (he claims the crew made it to the skyline before noticing him missing). This teaching style permeated his classroom. He wrote all his own problems, and they seldom resembled the problems that could be found in a mathematics textbook. Just as he went overboard in his coast guard training, he went overboard in teaching us mathematics. I loved being challenged to solve his problems because of the pride I felt when I excelled.

I fondly remember the class we had to flip our notebooks horizontally to be able to write the full equation (see Figure 1). I recall a surge of pride for being the first to solve the problem correctly. His class was the only one that I had ever received a perfect score for on the state test. I do not think I realized at the time the depth of his impact on my trajectory as a future educator. Not only did he inspire me to be a teacher but my experience as a student in his class helped form the foundation for my deep-seated pedagogical beliefs, including those driving this research project. I held on to my notebook so I could one day utilize his rigorous questions to challenge my students. As I have gained more experience in the field and interacted with other educators it

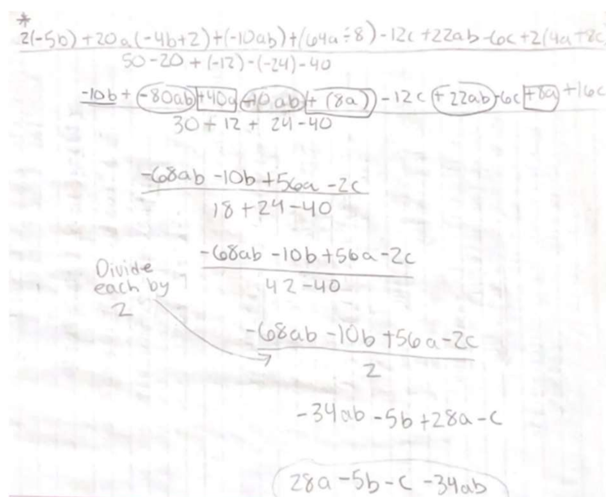


Figure 1. O'Connell's problem (own photo)

has become apparent and cemented in my heart that the way to improve all students' education is rooted in the growth of confidence that comes from overcoming challenges. Contemporary views seem to advocate for challenging students who are labeled as “gifted” or “high-level.” Meanwhile students who are viewed as being “low” are often not afforded the same opportunity to be challenged at an appropriate level. I got an especially bitter taste of this sentiment in one of my practicum placements.

The school I was in had a culture of classifying students by their ability levels as determined by a standardized test. During the placement, I was given the opportunity to plan a lesson. I designed a task that I felt challenged the students to create their own examples and questions. The morning I was supposed to teach the lesson, I felt the world drop out from under me when the teacher I was working with confided in me that they did not believe their “low-level” students were capable of coming up with their own examples. I was not upset that they did not like my idea, but I was devastated that they did not believe all students could rise to that challenge, even if some needed more support than others. They were unable to see beyond the arbitrary labels they had defined. Instead, we grouped students by their alleged ability levels and gave them problems that were at that level. My heart plummeted for the second time that day when I overheard one of the students remark that they were at the “dumb” kid table. These experiences affirmed my belief that appropriately challenging all students with tasks that require them to think is essential for learning and growth. This is not a belief that all students are at the same level or require the same challenges to grow. All students enter the classroom with different experiences and starting points, but tasks can be designed in ways that meet these varying needs while still challenging all students to think. To me, that is the definition of an equitable education, and it is exactly what we need to strive for. After a review of existing research in the field of academic tasks, cognitive demand, and productive struggle, I will develop a framework for planning lessons in a way to anticipate task implementation to not only maintain cognitive demand but differentiate productive struggle to provide a challenging and equitable education for all students. Further, I will provide an example lesson with my template to illustrate how it can be utilized.

Review of Literature

Background

Reform in education is not an emerging trend. However, the calls to reform mathematics instruction to promote thinking and reasoning skills attracted more attention in the 1980s and 1990s as it was emphasized by many educational organizations, such as the National Council for Teachers of Mathematics (NCTM). This also became the focus for many educational researchers including Walter Doyle, Mary Kay Stein, and Margaret Smith whose initiatives and studies are central to understanding existing work on academic tasks and cognitive demand. Despite mathematics education reform initiatives, “conventional mathematics instruction has placed a heavier emphasis on memorization and imitation than on understanding, thinking, reasoning, and explaining” (Silver & Stein, 1996, p. 478). A disconnect between reform goals and standard classroom instruction is conveyed.

Education reform often centers on changing the curriculum. In Doyle (1983) curriculum is defined as a “collection of academic tasks” (p.161). Defining curriculum in this way helps form an important link between curriculum and tasks, which is often missed when curriculum is viewed as the content that is supposed to be covered. By giving tasks a prominent role within the curriculum, Doyle is saying that it is as much about how the content is covered as it is about what is being covered. To alleviate the disconnect, it is not enough for curriculum reform to change the standards, but efforts must be made to change how the content is taught or the tasks that are being used. It makes sense that "the task you select and evaluate should match your goals for student learning" (Smith & Stein, 1998, p. 347). Fundamentally, to accomplish reform goals of promoting thinking and reasoning skills, the curriculum should consist of tasks that emphasize understanding, thinking, reasoning, and explaining.

Understanding the role tasks play within the curriculum alludes to a connection between tasks and thinking. Further, understanding the relationship between tasks and student thinking elicits a discussion about cognitive demand. Before being able to meaningfully consider how tasks and cognitive demand are related, the meaning of task and cognitive demand must first be clearly defined. Doyle (1983) provides a general definition of academic tasks as "the answers students are required to produce and the routes that can be used to obtain these answers" (p. 161). While Doyle's definition portrays a broader understanding of academic tasks, Stein and Smith's (1996) definition narrows specifically on mathematical tasks "as a segment of classroom activity that is devoted to the development of a particular mathematical idea" (p. 9). Both definitions hint at the active role tasks play in shaping learning.

Many believe that teachers play the most prominent role in learning, however, it is important to recognize that "teaching does not directly influence student learning but instead influences student thinking, which, in turn, influences their learning" (McCormick, 2016 p. 456). The impact teachers have on student thinking is largely derived from the tasks they select. Notably, students "will acquire information and operations that are necessary to accomplish the tasks they encounter" (Doyle, 1983, p. 162). Identifying this relationship between tasks and thinking solidifies the connection between learning and tasks. However, not all tasks promote thinking in the same way. Different kinds of tasks require different amounts of cognitive effort to solve. The distinction between different types of thinking provides the means to understand the relationship between tasks and cognitive demand. According to Doyle (1988), “the cognitive level of an academic task refers to the cognitive processes students are required to use in accomplishing tasks” (p. 170). Cognitive demand is a way to categorize tasks based on the cognitive level or amount of cognitive effort involved in successfully completing the task.

Categorizing Tasks According to Cognitive Demand

Determining how to distinguish between tasks that require different kinds of thinking is the first step for being able to conduct research into the cognitive aspects of tasks. Having the ability to understand differences in cognitive demand builds an understanding of the cognitive operations underlying different tasks, enabling the intentional selection of tasks to meet cognitive goals. The first distinction to be made is between low cognitive demand and high cognitive demand tasks. Low cognitive demand tasks require less cognitive effort. In other words, they do not require students to engage in complex cognitive processes. High cognitive demand tasks require more cognitive effort or complex processes to solve. Stein and Smith (1998) further classify tasks into 4 distinct categories: memorization, procedures without connections, procedures with connections, and “doing mathematics.”

Memorization tasks or procedures without connections are often viewed as subcategories of low cognitive demand tasks. Memorization tasks are “routine exercises that involve the memorization of formulas, algorithms, or procedures, and without connection to the underlying concepts or meaning” (McCormick, 2016, p. 3). Procedures without connections are “tasks that are algorithmic and focus solely on describing the procedure that was used” (McCormick, 2016, p. 3). Both definitions reveal similarities in the cognitive processes required to complete memorization or procedures without connection tasks. They are lower in demand because students are often engaging in rote learning of knowledge. For example, an activity requiring students to repeatedly practice the Pythagorean Theorem to memorize the formula and demonstrate the ability to successfully use the procedure to find missing side lengths of right triangles. Students would develop an understanding of what the Pythagorean Theorem is and how to use it, without understanding why it works, or perhaps even why it is important.

On the other hand, procedures with connections or “doing mathematics,” tasks are considered high cognitive demand. Procedures with connections are “tasks that focus on the use of broad general procedures for developing deeper understanding of concepts and ideas” (McCormick, 2016, p. 3). Lastly, “doing mathematics” are “tasks that require complex and non-algorithmic thinking to explore and understand the nature of mathematical concepts, processes, and relationships” (McCormick, 2016, p. 3). These kinds of tasks develop a greater understanding of mathematics by requiring students to engage in higher level cognitive processes, such as analyzing, synthesizing, and reflecting. If a teacher wanted to not only build an understanding of what and how to use the Pythagorean Theorem, but also why it works, they might select an activity that encourages students to “discover” the Pythagorean Theorem by contemplating the relationship between the leg lengths and the length of the hypotenuse of right triangles. Using the example of the Pythagorean Theorem, illustrates how tasks within different categories may cover the same content in vastly different ways just by altering the cognitive demand of the tasks. It is evident that understanding the different categories for classifying tasks by cognitive demand is imperative to effectively selecting tasks that promote higher order thinking.

Prior research has focused on developing teachers' ability to identify and correctly classify tasks into these categories, a prerequisite to being able to select and use high cognitive demand tasks in the classroom. Without knowledge of the different kinds of cognitive demands, it would be difficult to distinguish between low and high demand activities and understand how the selection of different tasks affect students' thinking.

Professional development is one avenue to support this acquisition of pedagogical knowledge. Stein, Smith, and Henningsen developed a task-sort activity with 20 tasks and a task-analysis guide (see Figure 2) that served as a scoring rubric for cognitive demand. The task-analysis guide outlines characteristics for different kinds of tasks to assist educators in the process of identifying and classifying tasks according to their cognitive demands. Their task sort and rubric have been used in a variety of settings to help educators develop this skill. Another initiative, by Boston and Smith, sought to use professional development to improve teachers' selection and use of high cognitive tasks. Their data indicated that "following their participation in the professional development initiative, project teachers more frequently selected high-level tasks as the main instructional tasks in their classrooms and had improved maintenance of high-level cognitive demands" (Boston & Smith, 2009, p. 119). The results indicate that by building teachers' awareness and knowledge of the different types of tasks,

Levels of Demands

Lower-level demands (memorization):
- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):
- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):
- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):
- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

Figure 2. The task-analysis guide (Source: Smith & Stein 1998, p. 348)

teachers are able to select high cognitive demand tasks with intention, therefore increasing their ability to use these kinds of tasks in their classrooms.

Nature of Tasks

In addition to having an awareness of the different categories, teachers must become familiar with the nature of the different kinds of tasks. In other words, it is important that they understand what task features influence cognitive demand. The task-analysis guide provided by Stein, Smith, and Henningsen offers a starting point for understanding these characteristics, especially the traits distinguishing low level tasks from high level tasks (see Figure 3). Task features to consider include the use of multiple solution strategies, the use of multiple representations, and requirements for students to explain their thinking.

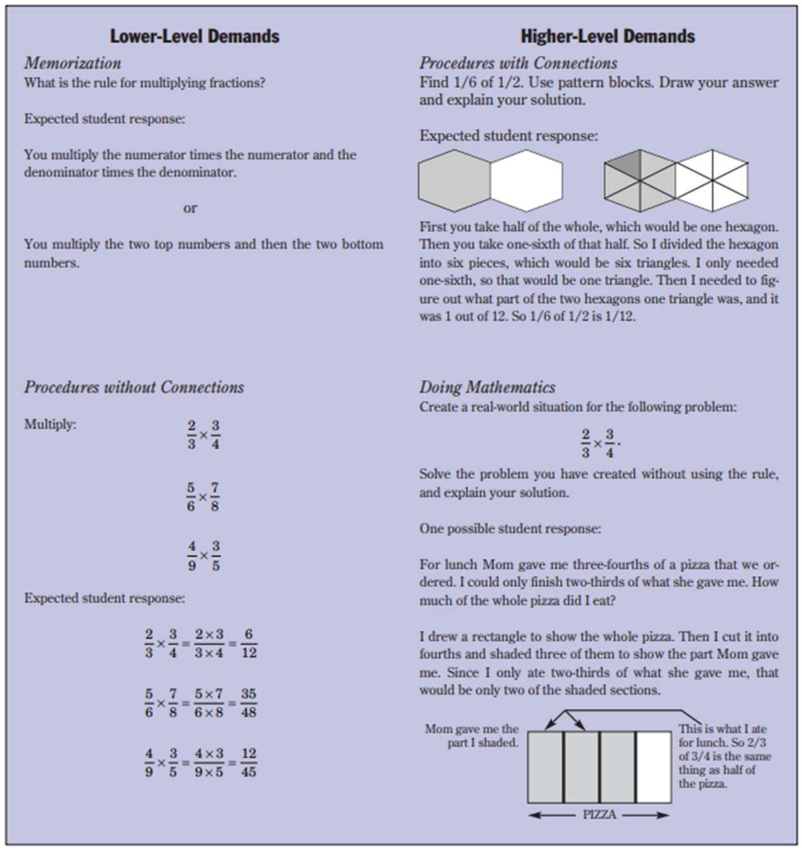


Figure 3. Examples of tasks at different levels of cognitive demand (Source: Smith & Stein, 1998, p. 349)

High cognitive demand tasks are "characterized by features such as having more than one solution strategy, as being able to be represented in multiple ways, and as demanding that students communicate and justify their procedures and understandings in written and/or oral form" (Stein, Grover, & Henningsen, 1996, p. 456). When a task exhibits those characteristics, it indicates a high cognitive nature. Lower demand is often accompanied by less ambiguity and more emphasis on one particular answer and/or procedure. Therefore, low cognitive demand tasks are less likely to incorporate the use of multiple solution strategies, multiple representations, and the production of explanations for student thinking. In order to purposefully select tasks based on cognitive demand, it will be important to consider these features as indicators.

The Appropriateness of Selecting High Cognitive Demand Tasks

To determine if a task is at an appropriate level of cognitive demand, teachers must "consider the students—their age, grade level, prior knowledge and experiences—and the norms and expectations for work in their classroom" (Smith & Stein, 1998, p. 344). Inherent differences exist between distinct groups of students. It follows that what will be an appropriate task for one group of students will not necessarily be the right task for other groups. Consequently, familiarity of the characteristics of students within a class is a central consideration when determining whether a task will be appropriate for those students.

It is evident that "in broad terms the curriculum of the early elementary grades reflects an emphasis on fundamental operations in reading and mathematics, the so-called "basic skills" (Doyle, 1983, p. 160). Therefore, it makes sense that "most elementary arithmetic skills are "learned" by rote memorization and assessed on a test of memory recall" (Willis, 2010, p. 8). The curriculum of elementary school reflects what is developmentally appropriate for that age group. However, "as students' progress through the grades, the emphasis gradually shifts from basic skills to the content and the methods of inquiry embodied in academic disciplines" (Doyle, 1983, p. 160). Notably, "in the middle school or junior high school years, students begin to develop the capacity for formal operational thought, that is, the ability to think abstractly and use general strategies to analyze and solve problems" (Doyle, 1983, p. 160). If considerations of the age group and grade level are taken into account, it becomes apparent that older students should be provided opportunities to engage in higher levels of thinking. This means that as students move into middle school, it is developmentally appropriate for students to be given high cognitive demand tasks.

The benefits of high cognitive tasks seem to be widely recognized by educational organizations and researchers. One of the biggest research studies into the impact of high cognitive tasks, The Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) study conducted by Silver and Stein (1996), found "student learning gains were especially positive in classrooms that could be characterized by the set up and implementation of instructional tasks that encouraged high-level thinking and reasoning and the use of multiple solution strategies, multiple connected representations, and mathematical explanations" (p. 506). Significantly, their research specifically focused on urban populations, with high numbers of economically disadvantaged students that were often considered low achieving. The premise for the study was based on the belief that their lower performance was "not due primarily to a lack of student ability or potential but rather to a set of educational practices that fail to provide them with high-quality mathematics learning opportunities" (Silver & Stein, 1996, p. 477). These practices are still pervasive in many classrooms despite "consistent recommendations for the exposure of students to meaningful and worthwhile tasks, tasks that are truly problematic for the students rather than simply a disguised way to have them practice an already-demonstrated algorithm" (Stein, Grover, & Henningsen, 1996, p. 456).

Many classrooms are still structured in a way that calls for direct instruction of content before students are provided with opportunities to practice. In these cases, challenging tasks are often reserved for "high level" students to work on as an extension, following their regular coursework. Further, challenging tasks are often viewed as inappropriate for "low-level" students, whereas direct instruction is often held as the most appropriate way to build their content knowledge. Lack of challenging instruction to these students may be due in part to "a pernicious belief that high-level mathematical objectives and performance expectations are not appropriate for all students," especially for students "who have experienced previous difficulty learning mathematics or who are otherwise assigned to "lower track" instruction" (Silver & Stein, 1996, p. 479). Yet, "evidence from several sources suggests that such learners have a production rather than capacity deficiency (Doyle, 1983, p. 175)." To reiterate, these students do not have a lack of ability or potential to think at higher levels. They deserve the opportunity to participate in meaningful and challenging tasks and to be afforded the same learning opportunities as their peers. This is not to say that they may not require different support or levels of challenge. Rather, the argument being made is that all students can benefit from an appropriate level of challenge that is differentiated to meet their needs.

For some teachers to recognize the importance of using these tasks to benefit all students in their classroom, they, "will need opportunities to thoroughly overhaul their thinking about what it means to know and understand mathematics, the kinds of mathematical tasks in which their students should engage, and how they can support their students' learning without taking over students' opportunities for high-level thinking and reasoning" (Boston & Smith 2009, p. 120). At this point, if the goal is to encourage students to engage in high levels of thinking and reasoning, then high cognitive demand tasks should be recognized as the type of tasks that teachers should strive to incorporate into their lessons. These tasks align with curriculum goals to promote the desired thinking and reasoning skills since students are required to engage in complex cognitive processes to complete these tasks. When engaged in high cognitive demand tasks, "students need to impose meaning and structure, make decisions about what to do and how to do it, and interpret the reasonableness of their actions and solutions" (Stein, Grover, & Henningsen, 1996, p. 456). However, it is important to note that the role of low cognitive demand tasks is not being diminished. The suggestion is not that high cognitive demand tasks are always most appropriate for learning, rather that they can be used to benefit all students' education by encouraging higher cognitive processes. Outside the scope of this paper are instances when low cognitive tasks might be more appropriate for learning. While selection of high cognitive demand tasks is an essential component, selection of a task is only one phase in the classroom through which a task passes. Building understanding of The Mathematical Task Framework, developed by Stein and Smith, creates a more complete view of tasks within instruction.

The Mathematical Task Framework

The Mathematical Task Framework (see Figure 4) is a structure describing the transformation a task might take through 3 phases of use within a classroom. The three phases are Task in Curriculum, Task as Planned, and Task as Implemented. The Framework once again emphasizes that student learning is the result of

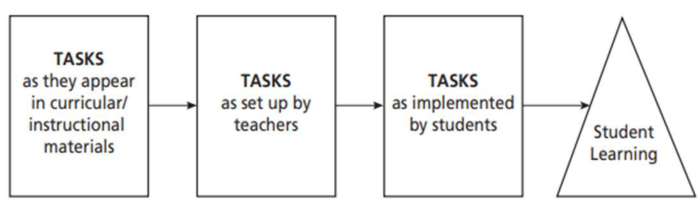


Figure 4. The mathematics task framework (Source: Stein & Smith, 1998, p. 11).

the kinds of tasks used in the classroom. It is important to note that due to the potential for task features to vary at different stages, cognitive demand is not static, rather it is subject to change.

The first phase is Task in Curriculum, which calls on teachers to recognize whether the task they are selecting from the curriculum contains the features of high cognitive tasks, discussed previously in this paper. Does the selected task utilize multiple solution strategies, multiple representations, and require students to explain their thinking? Developing the ability of teachers to be able to determine the cognitive demand of the tasks they are selecting from the curriculum based on the presence or absence of high cognitive features is just one component of using these tasks in the classroom.

The next phase, Task as Planned or Task as Set-Up, addresses whether the task is set up by the teacher for students to engage in high levels of thinking. Teachers affect learning during the set-up phase "by defining and structuring the work students do, that is, by setting specifications for products and explaining the processes that can be used to accomplish work" (Doyle, 1988, p. 169). During this phase cognitive demand specifically refers to the "kind of thinking processes entailed in solving the task as announced by the teacher" (Stein, Grover, & Henningsen, 1996, p. 461). The cognitive demand is affected by the presence or absence of the features in the task as planned. Is the task set-up to encourage multiple solution strategies, multiple representations, and the explanation of student thinking? The Set-Up phase includes "verbal directions, distribution of various materials and tools, and lengthy discussions of what is expected" (Stein, Grover, & Henningsen, 1996, p. 460). The planned task can look different than the task contained in the curriculum. It is within the teachers control to make decisions during planning regarding these features, which may result in shifts of cognitive demand. The demand may be maintained if these features are consistently utilized from curriculum to planning. Yet, a decline in demand may be seen if the task within the curriculum has these features but it is planned in a way that diminishes the features of high cognitive tasks. For example, if a task is designed in the curriculum to allow students to solve it in a variety of ways, but the teacher decides to provide guidance and instruction towards the solution that makes the most sense to them. The multiple solution strategies attributed to the task in the curriculum would no longer apply to the task as planned by the teacher. It is possible, but highly unlikely that a task could increase in demand if a teacher adds high cognitive features during planning to a task that did not otherwise have them. This would entail taking a task from the curriculum that did not have the features of high cognitive tasks and planning in such a way that the task would involve multiple solution strategies, multiple representations, and student explanations of thinking.

Following the Task Set-Up phase is the Implementation phase. This phase is concerned with the type of thinking students engage in when completing the task (Stein, Grover, & Henningsen, 1996). Do students use multiple solution strategies, multiple representations, and produce explanations of their thinking? The selection and planning of a high cognitive task is not enough to ensure student engagement at high levels without proper implementation practices to support the task. Again, demand may not stay at the intended level from curriculum or planning to implementation. Cognitive demand can be maintained or declined during implementation, but once again is unlikely to increase. For instance, a task within the curriculum or as planned by the teacher may encourage students to explain their thinking, but during the lesson, demand declines due to students pressing the teacher to explain how to solve the problem.

Oftentimes, changes in demand will occur between two successive phases of the framework. For example, “studies have found differences between the objectives of curricular materials and the ways in which teachers have interpreted and set up the material” (Stein, Grover, & Henningsen, 1996, p. 460). This illustrates a change that may occur from tasks contained within the curriculum to the task that is set up. The teachers' understanding of the content knowledge influences how they might choose to use curriculum materials. It is important to recognize how cognitive demand might change between the Curriculum and Planning phase, but for the purpose of this project, more emphasis will be placed on the shifts that occur between Planning and Implementation. It is difficult to implement high cognitive tasks in a way that maintains high levels of demand. Research shows a tendency of tasks designed at a higher level to decline since tasks are often transformed into familiar, procedural problems that are easier for teachers to implement. In the study conducted by Boston and Smith (2009) on the selection and enactment of high cognitive tasks, "no student-work tasks coded as low level (i.e., a score of 1 or 2) for Potential were subsequently coded as high-level for Implementation" (p. 140). Similar to how professional development was used to increase teachers' ability to classify tasks based on knowledge of the characteristics, professional development has been used as an avenue to increase awareness of features affecting implementation. Boston and Smith (2009) recruited teachers to participate in their Enhancing Secondary Mathematics Teacher Preparation (ESP) project. They found that "following participation in ESP, the ESP teachers were more likely to exhibit classroom factors that maintained high-level cognitive demand and less likely to exhibit classroom factors that reduce high-level cognitive demand" (Boston & Smith, 2009, p. 142). Although the factors that affect implementation are complex, the finding indicates that building awareness of the factors assists teachers in implementing high cognitive tasks more effectively.

This framework provides a powerful tool for teachers to be able to reflect on how the factors impact or might impact changes of demand between phases, therefore, informing instructional practices. Stein and Smith (1998) provided examples of how teachers used the framework to reflect. One participant, Theresa, planned a high-level task that declined in demand during implementation. When reflecting, “the framework gave her a language for describing events that had occurred in her classroom and for understanding why things may not have worked out as she had envisioned that they would,” as she “realized that the students’ lack of prior experience with open-ended tasks made them uncomfortable when they were presented with a task that they did not immediately know how to solve” (Stein & Smith, 1998, p. 12). By thinking about her lesson in terms of the Framework, she recognized how she came to dominate the thinking of the task during implementation. Similarly, another participant, Ron Castleman observed a decline in the demand of his procedures with connections task when he was pressured into providing guidance that surmounted student thinking and “divorced their thinking from the diagram and consequently from the meanings of decimal, percent, and fraction” (Stein & Smith, 1998, p. 12). After reflecting, “he then realized that he had contributed to their departure from the diagram by stepping in and suggesting that they start with the fraction” (Stein & Smith, 1998, p. 13). The framework made it possible for these teachers to reflect on the reason the cognitive demand of their tasks changed. Within this project, the Mathematical Task Framework builds an awareness of where changes in demand may occur and offers the opportunity to reflect on the factors affecting demand between the planning and implementation phase, which will enable the implementation phase to be anticipated during planning.

Factors of Task Implementation

Understanding factors affecting shifts in demand is key to planning for implementation. A list of factors "was derived from a study of nearly 150 tasks that had been used over a three-year period at four different schools" (see Figure 5). Some factors are associated with the maintenance of demand while others are associated with a decline in demand. Changes in task features that occur between phases, as previously discussed, is one of the many factors. In addition, scaffolding, building off prior knowledge, self-monitoring, modeling of high-level performance, emphasizing student thinking, and the formation of conceptual connections are other factors concerning the maintenance of demand during implementation.

Factors Associated with the Maintenance of High-Level Cognitive Demands

1. Scaffolding of student thinking and reasoning is provided.
2. Students are given the means to monitor their own progress.
3. Teacher or capable students model high-level performance.
4. Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback.
5. Tasks build on students' prior knowledge.
6. Teacher draws frequent conceptual connections.
7. Sufficient time is allowed for exploration—not too little, not too much.

Factors Associated with the Decline of High-Level Cognitive Demands

1. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem).
2. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.
3. Not enough time is provided to wrestle with the demanding aspects of the task, or too much time is allowed and students drift into off-task behavior.
4. Classroom-management problems prevent sustained engagement in high-level cognitive activities.
5. Task is inappropriate for a given group of students (e.g., students do not engage in high-level cognitive activities because of lack of interest, motivation, or prior knowledge needed to perform; task expectations are not clear enough to put students in the right cognitive space).
6. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade).

Figure 5. List of factors associated with maintenance or decline of cognitive demand (Source: Stein & Smith, 1998, p. 14).

Scaffolding facilitates different challenge levels "by providing a sequence of prompts or intermediate supports in content, materials, or teacher guidance" (Willis, 2010, p. 28). For example, a scaffolded support might entail providing a visual along with a word-problem. Some students may need more scaffolding to be successful with challenging tasks than others, but it is crucial to consider how support is being provided. Proper support, which maintains the cognitive demand of a task, does not take over the thinking but helps build on a student's prior knowledge so they can engage with the task.

Prior knowledge is another factor affecting the cognitive demand of a task as students rely on what they already know when faced with new tasks. Therefore, it is vital to utilize tasks that enable students to pull from a variety of backgrounds. Tasks of this sort are known as having a low entry point, meaning students with diverse levels of prior knowledge can still access the task.

The encouragement of self-monitoring or giving students the means to monitor their own progress helps maintain demand by drawing students' attention to their thought processes and understandings. Self-monitoring puts responsibility for learning on the students. By reaffirming the active role of students in the learning process, demand is maintained.

During class discussions, the modeling of high-level performance promotes deeper understanding by inviting students to expose themselves to the thought processes of their peers and solution strategies that may differ from their own. In other words, the students learn from each other. Demand is once again maintained by the active role and responsibilities of students in the learning process, rather than a focus on direct instruction from the teacher. Exposure to a variety of solution methods also fosters more conceptual connections as students draw comparisons between different methods. Conceptual connections build on prior knowledge to expand and deepen students' understanding for future challenges. The major takeaway from all these factors is that demand is maintained when the emphasis is placed on the role of students and student thinking within the classroom instead of on the explicit role of the teacher in instruction.

Allotment of time for students to engage with the challenging aspects of a task must be carefully considered to determine whether demand was maintained or declined. If students are not given adequate time, the task often declines in demand as the teacher is pressured to compensate by taking a more active part in directing student thinking. Imagine students are tasked with researching statistics to develop scatter plots showing different trends in data. If students are not provided sufficient time to locate real-world data representing a negative slope, they may press the teacher to provide additional examples or help finding resources. Therefore, the students would no longer be thinking through the challenge themselves.

Considering the appropriateness of the selected task for a group of students is another important factor, previously discussed. The prior focus was on distinguishing between levels of schooling. However, that does not diminish the difference that one grade-level makes, or even differences existing between students at the same grade-level. Difficulty of a task is not universal, rather, "the subjective complexity of any task obviously depends on the age and ability of the learner" (Doyle, 1983, p. 172). The implication for cognitive demand means it is essential to start with a task that is selected and designed, sensitive to the specific needs of the students if the goal is to maintain engagement at a high level. If the task is too easy or difficult, there will be a corresponding decline in demand.

Other factors resulting in decline include classroom norms, routinization of tasks, an emphasis on correctness over thinking, and teacher dispositions towards addressing students' struggles. Classroom norms refers to the customary form of instructional practices a teacher follows. Further, classroom norms are the practices students are accustomed to. For example, whether instruction is typically direct or indirect. Direct instruction, "means that academic tasks are carefully structured for students, they are explicitly told how to accomplish these tasks, and they are systematically guided through a series of exercises leading to mastery" (Doyle, 1983, p. 173). When direct instruction is used, focus is on the teacher to guide students, rather than on "the central role of self-discovery in fostering a sense of meaning and purpose for learning academic content," attributed to indirect instruction (Doyle, 1983, p. 176). As a result, direct instruction is frequently viewed as a practice that lowers demand.

Another classroom norm involves how collaborative work is employed within a classroom. Are students encouraged to work together or is independent work standard? It is said that "from a cognitive perspective, it has been argued that students can construct knowledge and elaborate understandings through collaborative work that they would not so readily construct or elaborate via individual work" (Silver & Stein, 1996, p. 485). Working together, students can help each other think through problems without necessarily taking away from the challenging aspects of the task. During collaborative work, cognition is still directed by students, rather than the teacher, facilitating the maintenance of high-level demand.

The tendency of tasks to decline in cognitive demand has been well established by previous studies, demonstrating how difficult these tasks are to implement. A task that is designed and set-up at a high level will often experience a decrease in demand if the teacher intentionally or unintentionally implements the task in a way that it takes on the characteristics of a low level, routine task. This often occurs if teachers provide too much guidance on how students should solve the problem. There would be fewer solution strategies or representations of the problem observed as students are inclined to solve or represent the problem in the way their teacher influenced them to. In other words, this is one way a complex cognitive activity may become a familiar, straight-forward task.

Routinization of tasks also correlates to an emphasis placed on getting “the” correct answer rather than students thinking about the problem. By nature, routine tasks are going to be classified as low cognitive demand. Routine tasks effectively build a student’s ability to recall content knowledge, which lends itself to getting “the” correct answer. In contrast, high cognitive tasks are characterized by having multiple ways to solve or represent them and by promoting student thinking. When teachers prioritize correct answers over thinking, it is natural to expect a corresponding decline in demand inherent to the nature of the tasks and the thinking they foster. High cognitive tasks hold students accountable to their thinking, rather than focusing on the correctness of their solution. To maintain the demand of a task at a high level, it is important to sustain a press for students to provide explanations for their answers. However, during implementation, students may attempt to circumvent demand.

High cognitive tasks are viewed by many students as risky due to greater ambiguity. Many students fear making mistakes and therefore urge the teacher to provide more guidance or assistance. Demand is lowered when teachers respond by “redefining or simplifying task demands, softening accountability to reduce risks, or creating a highly familiarized task environment to smooth out possible workplace tensions in advance” (Doyle, 1988, p. 174). Many educators feel uncomfortable when their students struggle to grasp content, but providing too much guidance or assistance, takes away student accountability for thinking and therefore learning, which is essential to implement high cognitive tasks at the intended level. Maintaining high cognitive demand requires teachers to allow their students to engage in a struggle to understand, which benefits the learning process when the struggle is productive.

Productive Struggle

The extent to which teachers allow students to engage in productive struggle has been identified as a factor for the maintenance of cognitive demand during implementation. Productive struggle is defined as “a student's efforts to make sense of mathematics, to figure something out that is not immediately apparent” (Warshauer, 2014, p. 376). Defining productive struggle with an emphasis on student’s efforts, reiterates the benefits of selecting high cognitive tasks and implementing them in ways that student thinking remains central to the learning process. Notably, “high-level cognitive demand tasks have high potential for struggle precisely because they demand intellectual work” (Warshauer, 2014, p. 379). The inherent role of productive struggle within high cognitive tasks once again recalls the value of selecting these kinds of tasks if the objective is to challenge students. Further, if students are engaging in productive struggle, it is a good indicator that the cognitive demand has been maintained at a high level from planning to implementation, since productive struggle is unlikely to occur when demand is lowered.

Unfortunately, productive struggle is not universally accepted as a significant component of the learning process. In fact, “students’ struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms,” therefore “students’ struggles are not viewed as meaningful learning opportunities” (Warshauer, 2014, p. 376). When it comes to “teaching students who have difficulties learning mathematics, especially those receiving special education services, the struggle is often removed” (Lynch, Hunt, & Lewis, 2018, p. 196). In many traditional classrooms, challenging tasks are reserved as an extension activity for the students labeled as “high-level.” This practice fails to note how struggle can support thinking, even in cases where a student is deemed to be at a “low-level.” Hesitation to provide students seen as “low” with challenging tasks may be rooted in a fear that the task would be too difficult for these students to complete and therefore pose a detriment to their learning. However, productive “struggle does not mean “endless frustration” or “overly difficult” problems but problems within a student’s zone of proximal development” (Townsend, Slavit, & McDuffie, 2018, p. 218).

The theory surrounding zone of proximal development is largely derived from the works of psychologist, Lev Vygotsky. Vygotsky (1978) notably observed how “the capability of children with equal levels of mental development to learn under a teacher’s guidance varied to a high degree (p. 86).” In acknowledging this, Vygotsky is fundamentally reaffirming that students will have different cognitive needs. The variability in cognition exists due to the distinctions between developmental levels. Every student has functions that will fall within their actual development level and functions within their potential development level. A student’s actual development level involves skills that the student is able to perform independently, in other words, cognitive operations that have already matured. In contrast, their potential development level entails functions that are in the process of maturing. The zone of proximal development is defined by “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). The goal of selecting a task within a student’s zone of proximal development is to provide a task challenging students slightly beyond what they could do independently. This constitutes an appropriate level of challenge because it provides the opportunity for students to mature the cognitive operations that fall within their potential development level, by engaging in a productive struggle. By utilizing high cognitive tasks in ways that all students are appropriately challenged, through their zone of proximal development, it becomes apparent how productive struggle contributes to learning, rather than detracting from it.

The case of Ron Castleman illuminates the benefit of productive struggle. After utilizing the mathematical task framework to reflect on his lesson, Mr. Castleman made intentional modifications to put his students' thinking at the center of the activity and ensure a greater opportunity for them to engage in a productive struggle. For example, he was more mindful of the ways he provided guidance to not monopolize thinking. Following the improvements to his instruction, “he realized how much more students learned from working through a problem rather than being handed a procedure to follow” (Stein & Smith, 1998, p. 14). In being pushed to discover their own solution pathways, students gained more from the experience than they would have if they followed a process designated by their teacher. It is evident that “raising awareness that struggling to make sense of mathematics as a natural part of “doing mathematics” can contribute to students and teachers recognizing that this phenomenon is a valuable part of learning with understanding” (Warshauer, 2014, p. 395). Instead of being deterred from incorporating challenging tasks in instruction, teachers should capitalize on how challenging tasks can enrich education and lead to better learning outcomes, not just for a particular subset of students, but for all students.

Productive Struggle Framework

A discussion of the Productive Struggle Framework will foster a better understanding of what constitutes a productive struggle (see Figure 6). This framework was developed by analyzing trends and patterns of student struggle and teacher responses to struggle to provide a “tool to integrate student struggle into tasks and instructional practices rather than avoid or prevent struggle” (Warshauer, 2014, p. 375). There are four dimensions to the framework: the task dimension, student struggle dimension, teacher response dimension, and outcome dimension. The task dimension focuses on the cognitive demand of a task. In order for a productive struggle to occur, students must be engaged with high cognitive tasks, reaffirming the importance of selecting these kinds of tasks.

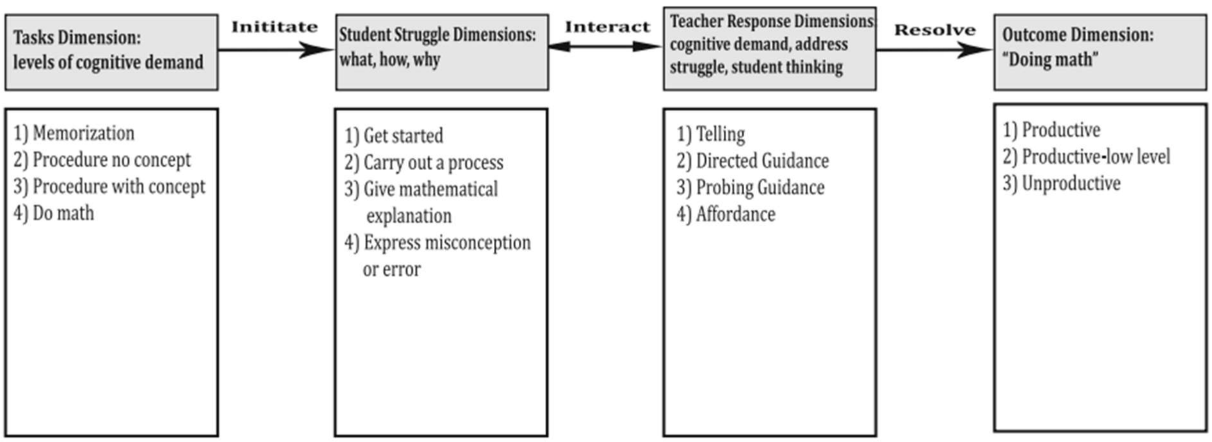


Figure 6. Productive struggle framework (Source: Warshauer, 2014, p. 391).

The Student Struggle Dimension provides insight on common instances of productive struggle. Productive struggle is likely to occur when students are getting started on the task, carrying out the task, giving the explanation for their thinking or expressing a misconception in their thinking. Struggles getting started are denoted by "voiced confusion about what the task asked them to do" (Warshauer, 2014, p. 384). This might occur, for instance, if students do not understand what a word problem is asking. When students struggle to carry out the process it can be a result of struggles to connect procedures to concepts or difficulties carrying out the procedure. An example of this would be if students do not understand that finding a constant rate in a word-problem implies using the slope formula or they mix up their "x" and "y" values when using the formula. When students are asked to explain, "students often struggle to verbalize their thinking and give reasons for their strategies even if their answer appeared correct on paper" (Warshauer, 2014, p. 386). Lastly, when "deep-seated mistaken ideas were used as a basis for solving problems" (Warshauer, 2014, p. 386), students often struggle to identify where they made an error. When pressed for explanations, students may respond "I don't know".

Struggles with getting started or carrying out the process are most likely to occur when students begin work individually, but can also occur during group work, while struggles explaining thinking or identifying errors are more likely to occur in group work settings but can occur individually. Understanding this dimension on a deeper level will enable educators to better anticipate when and how students struggle so they can better plan how they will respond. For example, teachers can plan how they will question students to guide their thinking or how they will direct group discussions to overcome these struggles.

The Teacher Response Dimension takes an even harder look into "the kind of guidance and structure teachers provide may either facilitate or undermine the productive efforts of students' struggles" (Warshauer, 2014, p. 376). Before being able to effectively plan responses to balance challenge and support of student struggles, educators must first develop a deeper understanding of the kinds of responses and their impact on learning. The 4 kinds of responses incorporated into the Productive Struggle Framework include telling, directed guidance, probing guidance, and affordance. With responses categorized as telling, "teachers generally provide sufficient information for the students to overcome the struggle" (Warshauer, 2014, p. 388). Telling responses parallel to direct instruction, such as "We are going to use x" (Warshauer, 2014, pg. 388). Demand is often removed as the task is transformed into procedures without connections. Similarly, directed guidance "redirects student thinking towards the teacher's thinking" (Warshauer, 2014, p. 388) and leads to "an answer built on the teacher's thinking rather than the student's" (Warshauer, 2014, p. 389). For example, "how do we go from this to a percentage?" (Warshauer, 2014, p. 338).

On the other hand, probing guidance and affordance typically maintain demand by putting the responsibility of resolving the struggle back on the students. Probing guidance helps resolve struggle by “consistently reverting to students’ thinking by building on their thinking and asking for explanations, reasons, and justifications” (Warshauer 2014, p. 389). Teachers use probing questions to ask for elaboration such as tell me what that means, tell me more, or by rephrasing the students thinking back to them. Affordance responses “provide opportunities for students to continue to engage in thinking about the problem and build on their ideas with limited intervention by the teacher” (Warshauer, 2014, p. 390). Examples of affordance responses include “why don’t you test it” and “what would that do for you?” (Warshauer, 2014, p. 390). In sum “by carefully questioning and listening to aspects of students’ struggles, the teacher can then make appropriate responses to build upon students’ ideas and thinking” (Warshauer, 2014, p. 380). The important distinction made in selecting an appropriate response to student struggle is that the focus should be on the student and their thinking.

The Outcome Dimension addresses whether a student struggle is productive or not. Ultimately, a struggle will be productive if teachers “(1) maintained the intended goals and cognitive demand of the task; (2) supported students’ thinking by acknowledging effort and mathematical understanding and (3) enabled students to move forward in the task execution through student actions” (Warshauer, 2014, p. 390). A deeper understanding of this framework should help empower teachers to afford students the opportunity to struggle in a productive manner and offer general insight into how that can be accomplished. Understanding how it can be done for all students within a classroom necessitates a discussion of how to differentiate productive struggle.

Differentiating Productive Struggle

It may seem counterintuitive, but intellectual growth stems from challenges that "fall within students' reasonable capabilities" (Warshauer, 2014, p. 377). Students traditionally viewed as low are no exception, however what falls within their reasonable capabilities might be different than their peers. Determining what is within a student's reasonable capabilities involves a precarious balancing of considerations of the student's zone of proximal development. A challenging task within the zone of proximal development does not surpass a student's readiness level to the extent that they are unlikely to be successful given proper supports but entails "an educational target just beyond what a student can do independently when provided appropriate support" (Townsend, Slavit, & McDuffie, p. 218). In other words, tasks within the zone of proximal development will, "require students to exert mental effort, performing a task that is just difficult enough to hold their interest but not so difficult that they give up in frustration" (Willis, 2010, p. 17).

The zone of proximal development is all about finding this middle ground of a challenge that is just difficult enough to meet an individual student's needs. Not all students enter the classroom with the same understanding and background. The appropriate amount of challenge to encourage a productive struggle will be different depending on each student and their zone of proximal development. Selecting tasks at the right level of difficulty, within each student's zone of proximal development, is imperative to being able to differentiate productive struggle in a way that leads to intellectual growth for all students, rather than a hindrance of learning. Once teachers effectively identify a student's zone of proximal development based on an assessment of prior knowledge, tasks can be differentiated in a way that all students gain access to the task, but differentiation alone is not enough to guarantee the maintenance of rigor.

Differentiation refers to the “process through which teachers can increase access to content by considering unique characteristics of students as they plan instructional experiences” (Lynch, Hunt, & Lewis, 2018, p. 196). Fundamentally a task can be modified through many means: the content (what the student is learning), the process (how the students access the information), the product (how students demonstrate learning) (Lynch, Hunt, & Lewis, 2018, p. 196), and learning environment (where learning occurs). Through these means, teachers can alter the task to create access according to different student needs. While there are many strategies to differentiate a task to increase access, not all of them also ensure that demand is maintained. Unfortunately, “attempts to increase accessibility can decrease the mathematical richness of the task” (Lynch, Hunt, & Lewis, 2018, p. 196). Consideration of the Aunt Martha’s Cupcakes task (see Figure 7) provides clarification of the distinction between appropriate differentiation strategies and those which result in a decline in demand during implementation.

Aunt Martha has 5 trays of cupcakes. She describes how they were arranged: There are 100 total cupcakes on the trays. The first and second trays have 52 cupcakes, the second and third trays have 43 cupcakes, the third and fourth trays have 34 cupcakes, and the fourth and fifth trays have 30 cupcakes. How many cupcakes are on each tray? (Bair and Mooney 2013, p. 326)

Figure 7. Aunt Martha's cupcakes problem (Source: Lynch, Hunt, & Lewis, 2018, p. 197).

Differentiation could mean supplying more support or guidance of the teacher’s solution pathway, but that would rob students of the opportunity to engage in productive struggle. For example, in differentiating the Martha’s Cupcakes task, “it may be tempting to provide these students with the total number of cupcakes that are on one of the trays” (Lynch, Hunt, & Lewis, 2018, p. 197). By giving part of the answer, the complexity of the task is reduced, and the solution strategy is simplified. This decreases the cognitive demand and results in fewer opportunities for productive struggle. Providing students with too much support or guidance is a common error when differentiating tasks. Other common errors when differentiating to maintain productive struggle include providing the formula or process, solely focusing on the procedure without contextual connections, and not considering how to differentiate the task for advanced students, in addition to struggling students.

On the other hand, appropriate differentiation, which supports the maintenance of cognitive demand and engagement in productive struggle, will employ strategies that supply students with the resources they need to think through the task. In other words, the goal is to use strategies to differentiate the task to increase access without removing cognitive rigor. Lynch, Hunt, and Lewis (2018) in their Accessible Practices framework expand on the strategies to support given by Warshauer (2015). In order to support productive struggle according to Warshauer (2015), it is important to ask purposeful questions geared towards student thinking, promote critical thinking (as opposed to focusing only on obtaining the correct answer), allow time for engagement in productive struggle, and establish classroom norms that acknowledge productive struggle as a component of learning. All of these strategies are necessary for the occurrence of productive struggle, but Lynch, Hunt, and Lewis (2018) notes how these strategies do not explicitly address differentiation.

They designed the Accessible Practices framework to address this gap. Their framework outlines appropriate differentiation strategies to meet all students' needs. The strategies include, identifying a clear mathematical concept, considering the prior knowledge required for successful engagement with the task, allotting sufficient time, identifying potential barriers or gaps in prior knowledge, considering feedback and questioning strategies that focus on student thinking, and the structure of classroom discussion. For example, differentiating the same task, the number of cupcakes can be adjusted, larger or smaller, depending on the readiness level of the students or students could be given an incorrect number of cupcakes, and asked to evaluate the plausibility of the solution. As an extension of the task, students could engage deeper with the concepts, by rewriting a similar problem.

In chapter 3 of *Learning to Love Math*, Judy Willis provides several more examples of how a high cognitive demand task can be designed and differentiated to provide low, medium, and high complexity variations of the task to meet a variety of students' needs without denying any students the opportunity to engage in productive struggle. Even at the lower levels of complexity, students are challenged appropriately. Her lesson on understanding division exhibits one example. Before breaking students into groups, she demonstrates a few examples of the division process using manipulatives. She then asks students to make predictions for the next demonstration. Their predictions were used to assess their prior knowledge. After providing some corrective feedback, she put students who were still getting incorrect predictions into group 1, students who started getting predictions correct into group 2, and students who seemed to already demonstrate a deeper understanding of division, based on this assessment, into group 3.

All the groups work towards the goal of "understanding the concept of division as a means of breaking larger quantities into specified numbers of portions and to recognize that the process is a tool for predicting how many objects will be in each new grouping" (Willis, 2010, p. 42). This goal is not altered for any of the groups, but appropriate adjustments are made to meet each group's needs in obtaining the goal. Group 1, the lowest complexity, develops their understanding of pre division skills through sharing activities. Group 2, the early conceptual thinking group, practice skills using monetary manipulatives and scenarios to determine how many objects they could buy at given costs. Group 3, the more abstract conceptual thinking group, already demonstrated an understanding of division with remainders. Like group 2, they work on problems about purchasing items, but with different objects at different prices. Their knowledge may be further expanded by having them work with the newspaper advertising section to analyze different sales or discussing the concept of remainders by developing a skit of the remainder as an animate object.

Throughout all the activities, there is a sustained press for students to explain their thinking. Supports are designed in a way to keep the emphasis on student thinking. The focus is not on the procedure of division, but building conceptual understanding of what is being done during the process of division. Not only are the needs of "struggling" students being met but there are also planned extensions at the right level of challenge for students who easily grasp the concept of division. Further, factors that maintain the cognitive demand are considered in the planning of this task, such as building the activity off prior knowledge, quality modeling, manipulatives, a clear learning target, and sustained press for student thinking but appropriate differentiation strategies are also clearly exhibited. The focus on student thinking and reasoning throughout these strategies distinguish them from the "common error" strategies discussed previously. While supporting productive struggle within the classroom is undoubtedly complicated, planning for differentiation enables educators to consider whether their actions will support productive struggle during implementation. Both Martha's Cupcake task and Willis's division lesson demonstrate how appropriate strategies can be planned before the lesson is taught, so the implemented lesson can be effectively differentiated for all students.

Planning For Implementation

The tendency of high cognitive tasks to decline in demand between phases of The Mathematical Task Framework poses a challenge to effectively using these kinds of tasks. Questions are raised about how to increase the likelihood that demand will be maintained from planning to implementation. It is helpful to analyze the reasons demand declines to reveal underlying factors driving the decreases in demand. It comes down to the fact that high cognitive tasks are harder to control and require a high degree of flexibility from the teacher as they respond in the moment. By anticipating the factors affecting cognitive demand during the planning phase, it seems reasonable that teachers regain some control of their lessons and put themselves in a better position to maintain demand for the given assignment. Researchers have explored this premise.

Margaret Smith, Victoria Bill, and Elizabeth Hughes suggest "one way to both control teaching with high-level tasks and promote success is through detailed planning prior to the lesson" (2008, pg. 133). They developed the Thinking Through a Lesson Protocol (TTLP) (see Figure 8) to illustrate how planning increases the chances of successfully implementing high-level tasks.

Fig. 2 Thinking Through a Lesson Protocol (TTLP)

PART 1: SELECTING AND SETTING UP A MATHEMATICAL TASK

What are your mathematical goals for the lesson (i.e., what do you want students to know and understand about mathematics as a result of this lesson)?

In what ways does the task build on students' previous knowledge, life experiences, and culture? What definitions, concepts, or ideas do students need to know to begin to work on the task? What questions will you ask to help students access their prior knowledge and relevant life and cultural experiences?

What are all the ways the task can be solved?

- Which of these methods do you think your students will use?
- What misconceptions might students have?
- What errors might students make?

What particular challenges might the task present to struggling students or students who are English Language Learners (ELL)? How will you address these challenges?

What are your expectations for students as they work on and complete this task?

- What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task?
- How will the students work—independently, in small groups, or in pairs—to explore this task? How long will they work individually or in small groups or pairs? Will students be partnered in a specific way? If so, in what way?
- How will students record and report their work?

How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task? How will you ensure that students understand the context of the problem? What will you hear that lets you know students understand what the task is asking them to do?

PART 2: SUPPORTING STUDENTS' EXPLORATION OF THE TASK

As students work independently or in small groups, what questions will you ask to—

- help a group get started or make progress on the task?
- focus students' thinking on the key mathematical ideas in the task?
- assess students' understanding of key mathematical ideas, problem-solving strategies, or the representations?
- advance students' understanding of the mathematical ideas?
- encourage *all* students to share their thinking with others or to assess their understanding of their peers' ideas?

How will you ensure that students remain engaged in the task?

- What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance in solving the task?
- What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?
- What will you do if a student (or group) focuses on non-mathematical aspects of the activity (e.g., spends most of his or her (or their) time making a poster of their work)?

PART 3: SHARING AND DISCUSSING THE TASK

How will you orchestrate the class discussion so that you accomplish your mathematical goals?

- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?
- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?
- What specific questions will you ask so that students will—
  1. make sense of the mathematical ideas that you want them to learn?
  2. expand on, debate, and question the solutions being shared?
  3. make connections among the different strategies that are presented?
  4. look for patterns?
  5. begin to form generalizations?

How will you ensure that, over time, *each* student has the opportunity to share his or her thinking and reasoning with their peers?

What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?

What will you do tomorrow that will build on this lesson?

Figure 8. Thinking through a lesson protocol (Source: Smith, Bill, & Hughes, 2008, p. 134).

Their protocol is broken down into 3 parts. First, selecting and setting up a mathematical task, next supporting student exploration of the task, and lastly sharing and discussing the task.

Breaking up planning in this manner distinguishes different components that need to be considered. Their first part focuses largely on identifying a clear mathematical concept, thinking about how the task could be solved in different ways, and setting class expectations, which all typically occur prior to a lesson. This is the most straight-forward section, composed of elements that are often already considered during planning but are nevertheless essential to support the learning process. The next component addresses student exploration. This section is where teachers speculate about actions occurring during implementation. Student actions create uncertainty for teachers using high cognitive tasks, but by designating a section to help teachers reflect on potential responses, some of the uncertainty can be mitigated by planning ahead. Lastly, teachers anticipate the class discussion around the task. This is another section that requires speculation but enables teachers to reflect on how they can utilize classroom discussions to promote learning. The significance of planning for class discussions is further demonstrated by the specialized attention it has received from researchers.

Another framework, the 5 Practices Model, focuses on facilitating discussions at a high level "to help increase the likelihood that the demands of high-level tasks will be maintained during instruction" (Smith, Hughes, Engle, & Stein, 2009, p. 550). By planning for discussions that are centered on student thinking rather than the teachers, the responsibility for thinking remains on the students, therefore maintaining demand. The first practice is to anticipate student responses which "also challenges teachers to understand the wide range of methods that a student might use to solve a task and think about how the different methods are related, as well as how to connect students' diverse ways of thinking to important disciplinary ideas" (Smith, Bill, & Hughes, 2008, p. 133). It allows teachers to think about the ways they provide feedback so they can respond in manners that will keep the focus on student thinking. The next practice is to monitor students' thinking while they work. As student work is monitored, the teacher can identify or select students to model work during the discussion. The teacher can strategically think about the strategies and students selected to formulate a better understanding of the intended concept. Purposeful decisions about the sequencing of strategies during discussions can also impact how students come to understand the concept. For example, starting with a more basic visual example and progressing to more complex, arithmetic solutions formulates deeper connections to the meaning of the procedure being taught. The last practice is to formulate connections. Once again, connections between different solution strategies often provide deeper insights into mathematical concepts. These works provide a foundation for developing The O'Connell Framework.

The O'Connell Framework

The goal of developing The O'Connell Framework (see Appendix A) is to provide a template for lesson planning that enables teachers to anticipate factors of implementation to better plan for the maintenance of high cognitive demand and differentiate the task in a manner that all students can engage in productive struggle at an appropriate level for their needs. It builds off the prior works discussed as far as planning for implementation, and then expands on how differentiation can also be planned prior to the lesson to provide all students the opportunity to engage in productive struggles.

The starting point for any lesson should be to identify standards to ensure that the task aligns with the content that needs to be taught (see Figure 9). The learning target will be derived from the standard, but provides a more concise statement about the goal for the lesson- what is the main concept all students should understand? The learning target will be taken into account when differentiating the activity because it is the target for all students, regardless of any additional support or extensions they may need. Before being able to differentiate or analyze the task, a brief description of the selected task needs to be supplied.

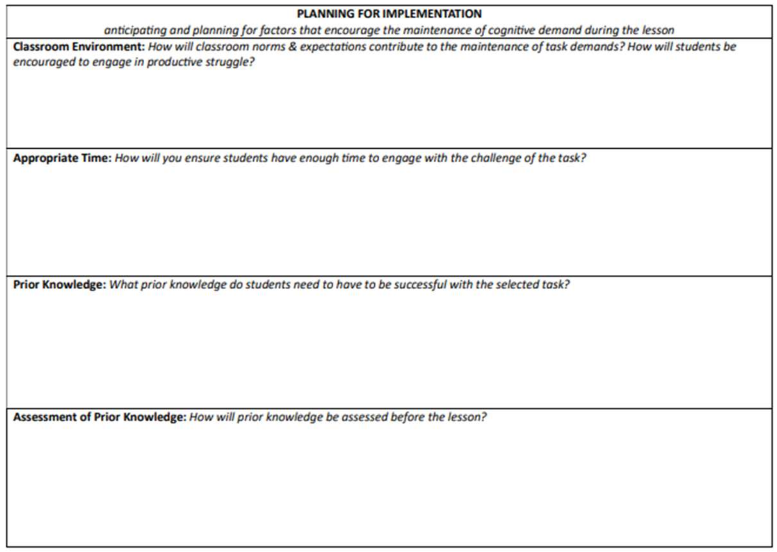
The O'Connell Framework

<table>
<tr><td colspan="2">Standard(s):</td><td colspan="4">Learning Target: what is the main concept you want <u>all</u> students to understand from the activity?</td></tr>
<tr><td colspan="6">Task: Briefly describe the selected task</td></tr>
<tr><td colspan="6">ANALYSIS OF SELECTED TASK
the main goal of this section is to confirm the nature of the selected task is high cognitive</td></tr>
<tr><td colspan="6">FEATURES OF HIGH COGNITIVE TASK
identify the features of the selected task</td></tr>
<tr><td colspan="2">☐ MULTIPLE SOLUTION STRATEGIES
Can the task be solved using a variety of solution strategies?</td><td colspan="2">☐ MULTIPLE REPRESENTATIONS
Can the task be represented in a variety of ways?</td><td colspan="2">☐ STUDENT THINKING
Does the task encourage students to explain their thinking/solution?</td></tr>
<tr><td colspan="6">TASK CLASSIFICATION
what category (or categories) best describe the selected task</td></tr>
<tr><td colspan="3">LOW COGNITIVE</td><td colspan="3">HIGH COGNITIVE</td></tr>
<tr><td colspan="3">☐ MEMORIZATION
The task involves routine exercises focused on the memorization of formulas, algorithms, or procedures.
☐ PROCEDURES W/O CONNECTIONS
Algorithmic tasks focused on the use of procedures.</td><td colspan="3">☐ PROCEDURES W/ CONNECTIONS
The task uses procedures to develop a deeper understanding of the learning target.
☐ "DOING" TASKS
The task focuses on complex thinking to explore and understand the learning target.</td></tr>
</table>

Figure 9. Page 1 of the O'Connell Framework (own photo).

Once the supporting information has been considered, the selected task undergoes an analysis. The main goal of this section is to confirm that the selected task is of a high cognitive nature. It is a critical component because if a task starts at a low level of cognitive demand, it has been established that it is highly unlikely for the demand to increase with implementation. In making sure the task is of high cognitive nature, first the task features must be analyzed. If the task has multiple solution strategies, multiple representations, and requires students to explain their thinking, these indicate a task of high cognitive nature. To further reflect on the selected task, it is helpful to think about how the task would be classified. Note that task categories are not mutually exclusive. A task may fit into multiple categories; however, it is vital to ensure that a task is predominantly procedures with connections, or "doing" tasks.

After the task has been analyzed to determine it is of a high cognitive nature, planning for implementation can be considered (see Figure 10). This section deliberates the factors identified as affecting the implementation of cognitive demand, so that they can be planned for, prior to teaching the lesson. Classroom Environment is the first aspect evaluated. Teachers are encouraged to think about how they can set up the classroom environment ahead of time in a manner that will contribute to the maintenance of demand. Considerations include classroom norms, expectations, and culture, as well as special consideration for how students will be encouraged to engage with the task and persevere through challenges. It is known that affording students adequate time to engage with the challenging aspect of the task is a factor of maintaining demand. Therefore, it is essential to plan to ensure enough time is allocated for engagement with the task. A critical component of student engagement is having the prior knowledge required to access the task and be successful with it. In other words, students draw on their prior knowledge when figuring out how to start working on a given task. Identifying what prior knowledge students will need to be successful serves a multitude of purposes. For one, it provides the opportunity to verify that the task selected will align with students' prior


PLANNING FOR IMPLEMENTATION
anticipating and planning for factors that encourage the maintenance of cognitive demand during the lesson

Classroom Environment: *How will classroom norms & expectations contribute to the maintenance of task demands? How will students be encouraged to engage in productive struggle?*

Appropriate Time: *How will you ensure students have enough time to engage with the challenge of the task?*

Prior Knowledge: *What prior knowledge do students need to have to be successful with the selected task?*

Assessment of Prior Knowledge: *How will prior knowledge be assessed before the lesson?*

Figure 10. Page 2 of the O'Connell Framework (own photo).

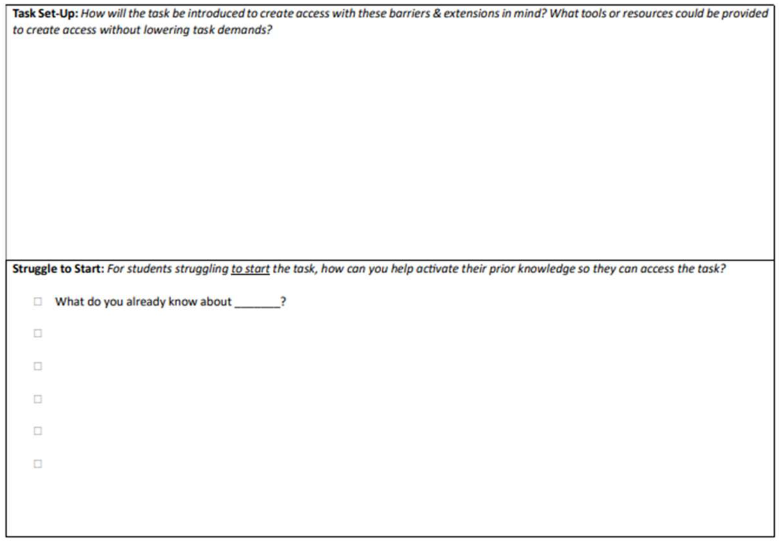
knowledge, which also ensures students have access to the task. A vital component is determining how prior knowledge will be assessed during the lesson so that the prior knowledge of students may be utilized in beneficial ways.

Significantly, reflecting on the assessment of prior knowledge helps identify gaps in prior knowledge that indicate a need for scaffolded supports, or alternatively, what understandings indicate the need for extensions (see Figure 11). In other words, the ability to effectively evaluate prior knowledge is a prerequisite for differentiating to meet the needs of all students. Based on the identification of potential barriers and indicators of the need for extension, appropriate, differentiated supports can be planned before the lesson. This enables the task to be implemented for all students to engage in productive struggle while developing an understanding of the learning target.

Identifying Barriers (Based on an Assessment of Prior Knowledge): *What gaps in understanding will indicate the need for scaffolded supports?*	**Identifying Needs for Extensions (Based on Prior Knowledge)** *What understandings will indicate the need for an extension of the task?*
☐ ☐ ☐ ☐	☐ ☐ ☐ ☐
Addressing Barriers *How can the task be differentiated to navigate barriers while maintaining demand?*	**Addressing Opportunities for Extensions** *How can the task be extended to further challenge students?*
☐ ☐ ☐ ☐	☐ ☐ ☐ ☐

Figure 11. Page 3 of the O'Connell Framework (own photo).

This leads directly to setting up the task. Setting up the task in a way that creates access for all students requires considering the barriers and extensions (see Figure 12). What tools or resources would prove valuable for creating access? In addition to contemplating tools or resources offered as support for students starting with the task, it is important to think about the manner in which feedback will be provided to students struggling to start the task. Struggling to start a task was identified as one of the key dimensions for student struggle by the Productive Struggle Framework. In planning to address this, teachers should think about what kinds of responses will be provided to encourage the students to reflect on their prior knowledge so they can access the task. For demand to be maintained, it is critical that any support offered does not monopolize student thinking.


Task Set-Up: *How will the task be introduced to create access with these barriers & extensions in mind? What tools or resources could be provided to create access without lowering task demands?*

Struggle to Start: *For students struggling to start the task, how can you help activate their prior knowledge so they can access the task?*

- ☐ What do you already know about _______?
- ☐
- ☐
- ☐
- ☐
- ☐

Figure 12. Page 4 of the O'Connell Framework (own photo).

Another dimension of student struggle involves undergoing the "process" of the task. To better anticipate how to respond to learners struggling with this dimension, teachers must first anticipate the possible solution strategies that students could use (see Figure 13). Based upon these anticipated strategies, teachers are able to target feedback in a way that guides students to evaluate their own thinking and persevere through the challenge of solving the problem. The goal is to ensure the students are afforded the chance to complete the problem in their own way and are not guided to a method designated by the teacher. Thinking about how these kinds of supports can be provided in response to possible solution strategies increases the likelihood that the teacher uses these kinds of responses, and therefore maintains the student's active role in solving the problem. Similarly, while thinking about possible solutions, inevitably, possible misconceptions or errors will be contemplated. It is important to also plan how to respond when these misconceptions or errors are observed during instruction. Again, the goal is for students to reflect on their own thinking, therefore it is often beneficial to ask questions that have students explaining or justifying their thought process. The goal is not to get students to recognize one "right" solution pathway.

POSSIBLE SOLUTION STRATEGIES FOR THE TASK AS GIVEN *anticipating possible solutions prior to implementation, enables the planning of teacher responses that will maintain demand*	
POSSIBLE SOLUTION STRATEGIES/REPRESENTATIONS *how can students go about solving the task?*	PLANNED TEACHER RESPONSES *how will you guide student thinking when they get stuck?*
	☐ ☐ ☐ ☐ ☐
POSSIBLE ERRORS & MISCONCEPTIONS *where might students make mistakes?*	PLANNED TEACHER RESPONSES *how can students be encouraged to reflect on their method? How can you guide students to recognize errors?*
	☐ ☐ ☐ ☐ ☐

Figure 13. Page 5 of the O'Connell Framework (own photo).

The final component of The O'Connell Framework involves planning for the discussion of the task in ways that will maintain high levels of student thinking (see Figure 14). It draws heavily on the 5 Practices Model for effectively using student responses in whole-class discussions. Of the possible solution strategies, which would be beneficial to have modeled in the discussion. If students are struggling to explain their thinking or struggle to identify an error, how will their thinking be prompted? Is there a certain order that responses should be modeled in to better build an understanding of the learning target? How do the solution strategies connect with each other? What do the connections reveal about the learning target? Planning for the implementation of the task in this way will restore a sense of control when it comes to using high cognitive tasks ensuring a better chance that cognitive demand can be maintained at a high level, and planning so the task can be differentiated in a way that does not deny any students of their right to engage in a productive struggle. An example lesson plan using this framework will be modeled to demonstrate its application.

SHARING & DISCUSSING THE TASK
Modeling: *What solution strategies should be modeled in the discussion?*
Questioning: *How can thinking be prompted when students struggle to explain their strategy?*
Sequencing: *In what order should the solution strategies be modeled?*
Connecting: *How do the solution strategies connect with each other? What do the connections reveal about the Learning Target?*

Figure 14. Page 6 of the O'Connell Framework (own photo).

Example Using the O’Connell Framework

Examining how the framework is used to plan for a lesson will foster familiarity with how this model can be used in practice (see Appendix B). The example, designs a lesson based on KY.6.G.1, a sixth-grade geometry standard (see Figure 15). From it, the learning target is derived. The intent is for all students to build an understanding of the relationship between the different geometric figueres and analyze how the relationships tie into area. No matter how the task is differentiated to better meet students’ needs, that is the understanding all students should develop from this task. The task has students explore and discover how to calculate the area of different shapes based upon the relationships of the shapes. It is confirmed that this task is of a high cognitive nature as it enables students to solve and represent their solutions in different ways and the emphasis is on students explaining their thinking, as opposed to getting the “answer.” When classifying this task, it falls under the categories of procedures with connections, because students are creating an understanding of why the “formulas” work, and “doing” mathematics, as students are guided to discover the formulas instead of being handed them.

The O’Connell Framework

<table>
<tr><td colspan="2">Standard(s):
KY.6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and quadrilaterals; apply these techniques in the context of solving real-world and mathematical problems</td><td colspan="4">Learning Target: what is the main concept you want all students to understand from the activity?
All students will build an understanding of how the relationships between squares, triangles, rectangles, and parallelograms relate to the areas of the different figures.</td></tr>
<tr><td colspan="6">Task: Briefly describe the selected task
Students will explore and discover the formulas for calculating the area of right triangles, rectangles, and parallelograms by fundamentally considering how the shapes relate to one another. They will transform a 4X4 square into the other shapes with equivalent areas (except the triangle) and analyze why the “formulas” make sense. (Task attached at the end)</td></tr>
<tr><td colspan="6">ANALYSIS OF SELECTED TASK
the main goal of this section is to confirm the nature of the selected task is high cognitive</td></tr>
<tr><td colspan="6">FEATURES OF HIGH COGNITIVE TASK
identify the features of the selected task</td></tr>
<tr><td colspan="2">☒ MULTIPLE SOLUTION STRATEGIES
Can the task be solved using a variety of solution strategies?</td><td colspan="2">☒ MULTIPLE REPRESENTATIONS
Can the task be represented in a variety of ways?</td><td colspan="2">☒ STUDENT THINKING
Does the task encourage students to explain their thinking/solution?</td></tr>
<tr><td colspan="6">TASK CLASSIFICATION
what category (or categories) best describe the selected task</td></tr>
<tr><td colspan="3">LOW COGNITIVE</td><td colspan="3">HIGH COGNITIVE</td></tr>
<tr><td colspan="3">☐ MEMORIZATION
The task involves routine exercises focused on the memorization of formulas, algorithms, or procedures.
☐ PROCEDURES W/O CONNECTIONS
Algorithmic tasks focused on the use of procedures.</td><td colspan="3">☒ PROCEDURES W/ CONNECTIONS
The task uses procedures to develop a deeper understanding of the learning target.
☒ “DOING” TASKS
The task focuses on complex thinking to explore and understand the learning target.</td></tr>
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Figure 15. Example of page 1 using the O'Connell Framework (own photo).

Next, it is important to plan for the implementation of the task, considering factors that will impact the maintenance or decline of demand (see Figure 16). Classroom environment is

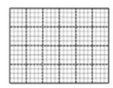
PLANNING FOR IMPLEMENTATION *anticipating and planning for factors that encourage the maintenance of cognitive demand during the lesson*
Classroom Environment: *How will classroom norms & expectations contribute to the maintenance of task demands? How will students be encouraged to engage in productive struggle?* Student collaboration will be encouraged to help each other think through the problems. Having peers as a resource, in some cases eliminates the need for teacher assistance, which will help keep thinking student-centered. Students also know that we prioritize thinking over "correct" answers. I have continuously worked to build a culture that is not afraid to get problems "wrong" as that is how we learn.
Appropriate Time: *How will you ensure students have enough time to engage with the challenge of the task?* I will monitor students' progress as they work. I will assess to determine whether the original timeline I had planned for the lesson is adequate for the majority of the students or if I need to designate more time for the activity. Further, for students who finish the assigned task, I will ensure I have extension activities in mind that enable them to continue thinking deeply about the relationships between shapes and area.
Prior Knowledge: *What prior knowledge do students need to have to be successful with the selected task?* The Coherent Standard, **KY.5.NF.4**: Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. b. Find the area of a rectangle with fractional side lengths by tiling it with squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. Students should have started building an understand of area as the space (i.e., number of tiles), a rectangle contains, which is equivalent to multiplying the lengths of the sides (number of tiles on one side multiplied by the number of tiles on the other side)
Assessment of Prior Knowledge: *How will prior knowledge be assessed before the lesson?* Class will be started will a warm-up question: How would you find the area of this figure? Based on this question, I could assess whether a student is more inclined (indicating they are more comfortable) counting the number of squares or if they readily recall being able to multiply the side lengths 

Figure 16. Example of page 2 using the O'Connell Framework (own photo).

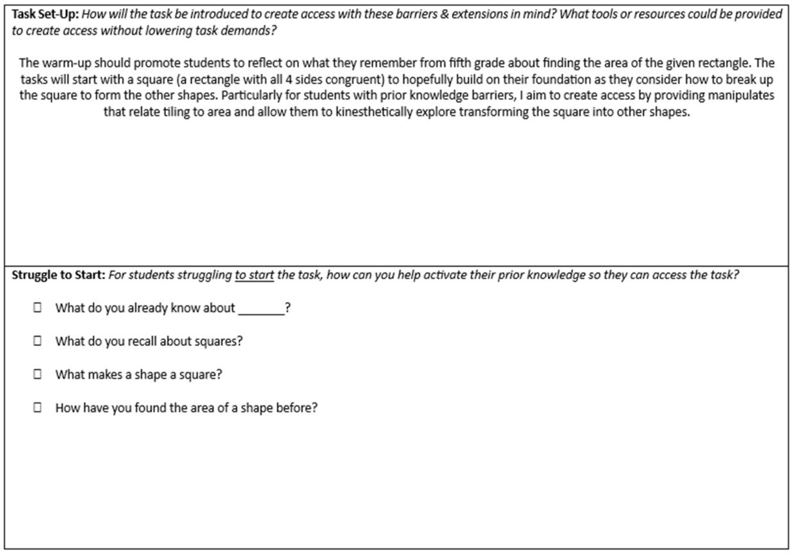
one big consideration. The goal is to create an environment focused on student thinking and perseverance through challenges. For example, an environment of collaboration, will open the possibility for students to rely on each other as a resource, in some cases eliminating the need for teacher interference, and maintaining the focus on student thinking. Further, if a classroom culture has been built that making mistakes is central to the learning process, it encourages students to persist with challenging tasks. As far as appropriate time, the main goal is to ensure that students are given the time to work through the challenges. This means alleviating the pressure of time constraints where possible, as this often precludes teachers taking over the thinking of a task. Essentially, teachers should remain flexible in their timelines and be willing to take more time for the activity if it is required. On the other end, teachers should make sure they have planned extensions of the activity, for students who complete the task quickly. Prior knowledge examines what students should already know from previous grades or lessons. In fifth-grade, students learn about the area of a rectangle as it relates to multiplying fractional side lengths. This pulls heavily on a visual understanding, as students connect area to the tiling of a figure. Understanding this concept will be essential for students understanding the relation of the areas of other figures, beyond rectangles according to the sixth-grade standard. Their understanding of the concept will be assessed through a bell-ringer having students explain how they would find the area of a given rectangle. It should become apparent whether students feel comfortable multiplying the side lengths to determine area, or if they still seem to rely on counting tiles.

Assessing prior knowledge enables educators to plan for how a lesson could be differentiated to meet a range of student needs (see Figure 17). Teachers can determine what gaps in prior knowledge will indicate the need for additional supports, and how the supports can be provided to sustain the challenge. Or alternatively, what knowledge will indicate the need for an extension activity, and how it can be provided. For this lesson, if students still demonstrate an inclination towards counting tiles to determine the area of the figure, it may indicate the need for additional support through this activity. Giving these students manipulatives that help them continue to formulate connections between the number of tiles and the area of the figure assists them in accessing the task, without taking away the challenge or modifying the learning target. Students who already indicate some understanding of the formula for finding area, or already demonstrate an understanding of shapes as composite figures will likely need extensions to better meet their needs. While the standard task aims for students to ultimately describe the process of finding the area for each shape, these students could be urged to contemplate how they could express the process as a formula. Another extension activity might give them a more complex, irregular composite figure to think about.

Identifying Barriers (Based on an Assessment of Prior Knowledge): *What gaps in understanding will indicate the need for scaffolded supports?*	**Identifying Needs for Extensions (Based on Prior Knowledge)** *What understandings will indicate the need for an extension of the task?*
☐ If during the warm-up students indicate an inclination to count the number of squares as opposed to multiplying side lengths ☐ Not readily making the connection between area as the amount of space (number of blocks)	☐ If students indicate a deeper understanding of multiplying the side lengths as a formula (b*h) ☐ If students already readily demonstrate an understanding of the relationships between shapes
Addressing Barriers *How can the task be differentiated to navigate barriers while maintaining demand?*	**Addressing Opportunities for Extensions** *How can the task be extended to further challenge students?*
☐ Students will be given manipulatives with tiling on the back to help them continue to formulate connections between counting tiles and multiplying sides in determining area	☐ Encourage students to think about how they could express the area of squares, triangles, and parallelograms in a similar way (writing the formulas- rather than expressing the process) ☐ Have students consider the area of an irregularly shaped composite figure.

Figure 17. Example of page 3 using the O'Connell Framework (own photo).

It is crucial to plan for how the task will be set-up prior to teaching the lesson (see Figure 18). This section enables teachers to reflect on how they will create access to the task,



Task Set-Up: *How will the task be introduced to create access with these barriers & extensions in mind? What tools or resources could be provided to create access without lowering task demands?*

The warm-up should promote students to reflect on what they remember from fifth grade about finding the area of the given rectangle. The tasks will start with a square (a rectangle with all 4 sides congruent) to hopefully build on their foundation as they consider how to break up the square to form the other shapes. Particularly for students with prior knowledge barriers, I aim to create access by providing manipulates that relate tiling to area and allow them to kinesthetically explore transforming the square into other shapes.

Struggle to Start: *For students struggling to start the task, how can you help activate their prior knowledge so they can access the task?*

- ☐ What do you already know about ______?
- ☐ What do you recall about squares?
- ☐ What makes a shape a square?
- ☐ How have you found the area of a shape before?

Figure 18. Example of page 4 using the O'Connell Framework (own photo).

particularly as students begin working on it. Introducing this activity with a bell-ringer will hopefully provide students a reminder of how they found the area of rectangles in fifth-grade. The task gets students to draw a connection between squares and rectangles, so they will be able to apply finding the area of a rectangle to finding the area of a square. Once again, manipulative would be an effective resource in creating access without undermining the challenge of the task. If students struggle to start the task, there are many ways to prompt them to reflect on their prior knowledge. For example, “what do you recall about squares?”, “What makes a shape a square?”, and “How have you previously found an area?”

In order to consider how other dimensions of student struggle could be addressed, it is beneficial to contemplate the variety of ways students may go about the problem and also areas where errors or misconceptions might be expected (see Figure 19). This enables targeted feedback to be planned in response to the anticipated struggles. Thinking through feedback ahead of time increases the chances that responses can be planned in ways to maintain demand. When students are observed counting tiles, a teacher could help facilitate deeper connections by asking the students if they can think of another way to arrive at the same answer. There are a variety of ways the original figure could be transformed. To ensure a deeper relationship is established between the figure, the teacher should have students elaborate on what they did to form the figure. This allows the teacher to determine if students understand what they are doing. On the other hand, students may struggle to recognize when areas are the same for different shapes. A question, such as, "how do we know when areas are the same?" affords students the opportunity to reflect on what it means to have the same area.

POSSIBLE SOLUTION STRATEGIES FOR THE TASK AS GIVEN *anticipating possible solutions prior to implementation, enables the planning of teacher responses that will maintain demand*	
POSSIBLE SOLUTION STRATEGIES/REPRESENTATIONS *how can students go about solving the task?*	PLANNED TEACHER RESPONSES *how will you guide student thinking when they get stuck?*
☐ Finding area: Student may multiply numbers or count squares. ☐ Creating figures with the same area ☐ Multiple ways to transform the figure into the other shapes	☐ When you observe students counting squares: Can you think of another way you could arrive at this answer? ☐ What do you think the area would be of ______? (ask a question to try to get them to apply their understanding in a different way) ☐ How did you know your figure had the same area? ☐ What did you do to get ______?
POSSIBLE ERRORS & MISCONCEPTIONS *where might students make mistakes?*	PLANNED TEACHER RESPONSES *how can students be encouraged to reflect on their method? How can you guide students to recognize errors?*
☐ All rectangles are squares. ☐ Knowing the areas are the same. ☐ Add side lengths or compare side lengths instead of area	☐ What does it mean to be a square? Can you think of any examples for dimensions of a rectangle that would not be a square? ☐ How do we know when areas are the same? (try to lead them to recognize that we use the same number of boxes to create the figures- except triangle which is half) ☐ What does it mean to find area?

Figure 19. Example of page 5 using the O'Connell Framework (own photo).

SHARING & DISCUSSING THE TASK
Modeling: *What solution strategies should be modeled in the discussion?* I anticipate parallelograms being the most challenging shape to form and understand since they are composites of rectangles and triangles. I want to ensure multiple representations of this are represented during the class discussion. As I am monitoring, student progress I will also be mindful of any unforeseen misconceptions to address and be aware of student who explain their thinking well (so they can share with the class)
Questioning: *How can thinking be prompted when students struggle to explain their strategy?* What are we trying to do/find? How do you think we might ________? What did that tell you about ________?
Sequencing: *In what order should the solution strategies be modeled?* Our discussion will follow the sequence of the handout. We will start with a discussion of squares, then triangles, then rectangles, and lastly parallelograms. I had made intentional decisions about this sequencing in designing the handout. To further solidify the connection between counting tiles and the process of finding the area, I will probably have students who effectively demonstrated use of the process explain first, then have a counting strategy represented to "double check."
Connecting: *How do the solution strategies connect with each other? What do the connections reveal about the Learning Target?* By having students work with shapes of the same area (except the square), I hoped to make it clearer, the connection between the different shapes. Ultimately, students should see that shapes that have the same area, will have the same number of "tiles." Throughout their transformations (except the square which is half), students should seem that they use the same number of blocks in different configurations.

Figure 20. Example of page 6 using the O'Connell Framework (own photo).

Class discussion is the last component to be planned (see Figure 20). Intentional decisions are made about modeling, questioning, sequencing, and connecting the tasks during discussions. As there will be a variety of ways to solve the problem, it is important to determine what strategies will benefit the class most and should therefore be modeled. Understanding the relationship between a rectangle and parallelogram is likely to be the most challenging for this task. As a result, presenting this concept in a variety of ways should be a priority of the discussion. When students struggle to explain their thinking in front of the class, there might be added pressure to compensate as they experience difficulty. However, it is just as important to maintain the demand of the task, and guide and support student thinking, rather than the teacher interjecting. Questions may start by having students address what the problem asks them to find. Other questions include "How do you think we might?" or "What did that tell you about?" In sequencing, it is best to follow the order of the handout when discussing the shapes. To better formulate connections, it is helpful to represent the tile counting strategy as well, but it makes the most sense to use that strategy to double check a more complex strategy underlying the process of finding an area. To further formulate connections between the shapes and their areas, students started with a four by four square and transformed it. By using the same number of blocks, except for construction of the triangle, it became clear that for shapes to have the same area, they must take up the same amount of space.

Conclusion

Every student deserves a Mr. O'Connell, a teacher that goes "overboard," for them, pushing them to grow because they believe in what the student is capable of accomplishing. It is an ambitious feat to aim to use high cognitive tasks to meet a wide variety of needs, but as shown through the development of The O'Connell Framework, is entirely possible with planning. To ensure students are engaging with the task at the intended level, it is critical to anticipate factors influencing the maintenance of demand when planning. Reimagining the way lessons can be planned, by shifting the burden of implementation and differentiation to planning, brings the goal of providing an equitable education for all students within reach for educators.

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Appendix A

The O'Connell Framework

Standard(s):	**Learning Target:** *what is the main concept you want* <u>*all*</u> *students to understand from the activity?*
Task: *Briefly describe the selected task*	

ANALYSIS OF SELECTED TASK *the main goal of this section is to confirm the nature of the selected task is high cognitive*		
FEATURES OF HIGH COGNITIVE TASK *identify the features of the selected task*		
☐ MULTIPLE SOLUTION STRATEGIES *Can the task be solved using a variety of solution strategies?*	☐ MULTIPLE REPRESENTATIONS *Can the task be represented in a variety of ways?*	☐ STUDENT THINKING *Does the task encourage students to explain their thinking/solution?*

TASK CLASSIFICATION *what category (or categories) best describe the selected task*	
LOW COGNITIVE	HIGH COGNITIVE
☐ MEMORIZATION *The task involves routine exercises focused on the memorization of formulas, algorithms, or procedures.* ☐ PROCEDURES W/O CONNECTIONS *Algorithmic tasks focused on the use of procedures.*	☐ PROCEDURES W/ CONNECTIONS *The task uses procedures to develop a deeper understanding of the learning target.* ☐ "DOING" TASKS *The task focuses on complex thinking to explore and understand the learning target.*

PLANNING FOR IMPLEMENTATION *anticipating and planning for factors that encourage the maintenance of cognitive demand during the lesson*
Classroom Environment: *How will classroom norms & expectations contribute to the maintenance of task demands? How will students be encouraged to engage in productive struggle?*
Appropriate Time: *How will you ensure students have enough time to engage with the challenge of the task?*
Prior Knowledge: *What prior knowledge do students need to have to be successful with the selected task?*
Assessment of Prior Knowledge: *How will prior knowledge be assessed before the lesson?*

Identifying Barriers (Based on an Assessment of Prior Knowledge): *What gaps in understanding will indicate the need for scaffolded supports?*	**Identifying Needs for Extensions (Based on Prior Knowledge)** *What understandings will indicate the need for an extension of the task?*
☐ ☐ ☐ ☐	☐ ☐ ☐ ☐
Addressing Barriers *How can the task be differentiated to navigate barriers while maintaining demand?*	**Addressing Opportunities for Extensions** *How can the task be extended to further challenge students?*
☐ ☐ ☐ ☐	☐ ☐ ☐ ☐

Task Set-Up: *How will the task be introduced to create access with these barriers & extensions in mind? What tools or resources could be provided to create access without lowering task demands?*
Struggle to Start: *For students struggling to start the task, how can you help activate their prior knowledge so they can access the task?* ☐ What do you already know about _______? ☐ ☐ ☐ ☐ ☐

POSSIBLE SOLUTION STRATEGIES FOR THE TASK AS GIVEN *anticipating possible solutions prior to implementation, enables the planning of teacher responses that will maintain demand*	
POSSIBLE SOLUTION STRATEGIES/REPRESENTATIONS *how can students go about solving the task?*	PLANNED TEACHER RESPONSES *how will you guide student thinking when they get stuck?*
	□ □ □ □ □
POSSIBLE ERRORS & MISCONCEPTIONS *where might students make mistakes?*	PLANNED TEACHER RESPONSES *how can students be encouraged to reflect on their method? How can you guide students to recognize errors?*
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SHARING & DISCUSSING THE TASK
Modeling: *What solution strategies should be modeled in the discussion?*
Questioning: *How can thinking be prompted when students struggle to explain their strategy?*
Sequencing: *In what order should the solution strategies be modeled?*
Connecting: *How do the solution strategies connect with each other? What do the connections reveal about the Learning Target?*

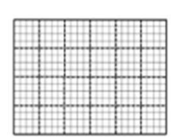
Appendix B

The O'Connell Framework

Standard(s): **KY.6.G.1** *Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and quadrilaterals; apply these techniques in the context of solving real-world and mathematical problems*	**Learning Target:** *what is the main concept you want <u>all</u> students to understand from the activity?* All students will build an understanding of how the relationships between squares, triangles, rectangles, and parallelograms relate to the areas of the different figures.
Task: *Briefly describe the selected task* Students will explore and discover the formulas for calculating the area of right triangles, rectangles, and parallelograms by fundamentally considering how the shapes relate to one another. They will transform a 4X4 square into the other shapes with equivalent areas (except the triangle) and analyze why the "formulas" make sense. (Task attached at the end)	

ANALYSIS OF SELECTED TASK *the main goal of this section is to confirm the nature of the selected task is high cognitive*		
FEATURES OF HIGH COGNITIVE TASK *identify the features of the selected task*		
☒ MULTIPLE SOLUTION STRATEGIES *Can the task be solved using a variety of solution strategies?*	☒ MULTIPLE REPRESENTATIONS *Can the task be represented in a variety of ways?*	☒ STUDENT THINKING *Does the task encourage students to explain their thinking/solution?*

TASK CLASSIFICATION *what category (or categories) best describe the selected task*	
LOW COGNITIVE	HIGH COGNITIVE
☐ MEMORIZATION *The task involves routine exercises focused on the memorization of formulas, algorithms, or procedures.* ☐ PROCEDURES W/O CONNECTIONS *Algorithmic tasks focused on the use of procedures.*	☒ PROCEDURES W/ CONNECTIONS *The task uses procedures to develop a deeper understanding of the learning target.* ☒ "DOING" TASKS *The task focuses on complex thinking to explore and understand the learning target.*

PLANNING FOR IMPLEMENTATION *anticipating and planning for factors that encourage the maintenance of cognitive demand during the lesson*
Classroom Environment: *How will classroom norms & expectations contribute to the maintenance of task demands? How will students be encouraged to engage in productive struggle?* Student collaboration will be encouraged to help each other think through the problems. Having peers as a resource, in some cases eliminates the need for teacher assistance, which will help keep thinking student-centered. Students also know that we prioritize thinking over "correct" answers. I have continuously worked to build a culture that is not afraid to get problems "wrong" as that is how we learn.
Appropriate Time: *How will you ensure students have enough time to engage with the challenge of the task?* I will monitor students' progress as they work. I will assess to determine whether the original timeline I had planned for the lesson is adequate for the majority of the students or if I need to designate more time for the activity. Further, for students who finish the assigned task, I will ensure I have extension activities in mind that enable them to continue thinking deeply about the relationships between shapes and area.
Prior Knowledge: *What prior knowledge do students need to have to be successful with the selected task?* The Coherent Standard, **KY.5.NF.4**: Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. b. Find the area of a rectangle with fractional side lengths by tiling it with squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. Students should have started building an understand of area as the space (i.e., number of tiles), a rectangle contains, which is equivalent to multiplying the lengths of the sides (number of tiles on one side multiplied by the number of tiles on the other side)
Assessment of Prior Knowledge: *How will prior knowledge be assessed before the lesson?* Class will be started will a warm-up question: How would you find the area of this figure? Based on this question, I could assess whether a student is more inclined (indicating they are more comfortable) counting the number of squares or if they readily recall being able to multiply the side lengths 

Identifying Barriers (Based on an Assessment of Prior Knowledge): *What gaps in understanding will indicate the need for scaffolded supports?*	**Identifying Needs for Extensions (Based on Prior Knowledge)** *What understandings will indicate the need for an extension of the task?*
☐ If during the warm-up students indicate an inclination to count the number of squares as opposed to multiplying side lengths ☐ Not readily making the connection between area as the amount of space (number of blocks)	☐ If students indicate a deeper understanding of multiplying the side lengths as a formula (b*h) ☐ If students already readily demonstrate an understanding of the relationships between shapes
Addressing Barriers *How can the task be differentiated to navigate barriers while maintaining demand?*	**Addressing Opportunities for Extensions** *How can the task be extended to further challenge students?*
☐ Students will be given manipulatives with tiling on the back to help them continue to formulate connections between counting tiles and multiplying sides in determining area	☐ Encourage students to think about how they could express the area of squares, triangles, and parallelograms in a similar way (writing the formulas- rather than expressing the process) ☐ Have students consider the area of an irregularly shaped composite figure.

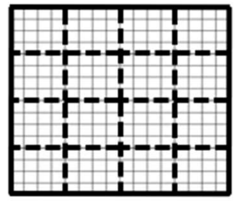
Task Set-Up: *How will the task be introduced to create access with these barriers & extensions in mind? What tools or resources could be provided to create access without lowering task demands?* The warm-up should promote students to reflect on what they remember from fifth grade about finding the area of the given rectangle. The tasks will start with a square (a rectangle with all 4 sides congruent) to hopefully build on their foundation as they consider how to break up the square to form the other shapes. Particularly for students with prior knowledge barriers, I aim to create access by providing manipulates that relate tiling to area and allow them to kinesthetically explore transforming the square into other shapes.
Struggle to Start: *For students struggling <u>to start</u> the task, how can you help activate their prior knowledge so they can access the task?* ☐ What do you already know about _______? ☐ What do you recall about squares? ☐ What makes a shape a square? ☐ How have you found the area of a shape before?

POSSIBLE SOLUTION STRATEGIES FOR THE TASK AS GIVEN *anticipating possible solutions prior to implementation, enables the planning of teacher responses that will maintain demand*	
POSSIBLE SOLUTION STRATEGIES/REPRESENTATIONS *how can students go about solving the task?*	PLANNED TEACHER RESPONSES *how will you guide student thinking when they get stuck?*
☐ Finding area: Student may multiply numbers or count squares. ☐ Creating figures with the same area ☐ Multiple ways to transform the figure into the other shapes	☐ When you observe students counting squares: Can you think of another way you could arrive at this answer? ☐ What do you think the area would be of ______? (ask a question to try to get them to apply their understanding in a different way) ☐ How did you know your figure had the same area? ☐ What did you do to get ______?
POSSIBLE ERRORS & MISCONCEPTIONS *where might students make mistakes?*	PLANNED TEACHER RESPONSES *how can students be encouraged to reflect on their method? How can you guide students to recognize errors?*
☐ All rectangles are squares. ☐ Knowing the areas are the same. ☐ Add side lengths or compare side lengths instead of area	☐ What does it mean to be a square? Can you think of any examples for dimensions of a rectangle that would not be a square? ☐ How do we know when areas are the same? (try to lead them to recognize that we use the same number of boxes to create the figures- except triangle which is half) ☐ What does it mean to find area?

SHARING & DISCUSSING THE TASK
Modeling: *What solution strategies should be modeled in the discussion?* I anticipate parallelograms being the most challenging shape to form and understand since they are composites of rectangles and triangles. I want to ensure multiple representations of this are represented during the class discussion. As I am monitoring, student progress I will also be mindful of any unforeseen misconceptions to address and be aware of student who explain their thinking well (so they can share with the class)
Questioning: *How can thinking be prompted when students struggle to explain their strategy?* What are we trying to do/find? How do you think we might _______? What did that tell you about _______?
Sequencing: *In what order should the solution strategies be modeled?* Our discussion will follow the sequence of the handout. We will start with a discussion of squares, then triangles, then rectangles, and lastly parallelograms. I had made intentional decisions about this sequencing in designing the handout. To further solidify the connection between counting tiles and the process of finding the area, I will probably have students who effectively demonstrated use of the process explain first, then have a counting strategy represented to "double check."
Connecting: *How do the solution strategies connect with each other? What do the connections reveal about the Learning Target?* By having students work with shapes of the same area (except the square), I hoped to make it clearer, the connection between the different shapes. Ultimately, students should see that shapes that have the same area, will have the same number of "tiles." Throughout their transformations (except the square which is half), students should seem that they use the same number of blocks in different configurations.

<u>Square:</u>

What makes a quadrilateral a square?

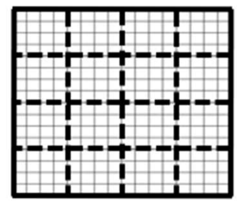


How do you find the area of a square?

What is the area of the given square?

<u>Triangle:</u>

How can we break the square into triangles?



What is the area of the largest triangle possible? How did you find the area?

What do you notice about the area of the largest triangle compared to the area of the square?

What does this tell us about the area of a triangle?

<u>Rectangle:</u>

If all squares are rectangles, are all rectangles, squares? Why?

How many rectangles can you think of that will have the <u>same</u> area as the given square? What are their dimensions?

What do you think would be the area of the following rectangle? Why?

What does this tell us about the area of a rectangle?

<u>Parallelogram:</u>

How can you create a parallelogram with the <u>same</u> area as the rectangle? What are the dimensions of your parallelogram?

What shapes make up a parallelogram?

What do you think would be the area of the following parallelogram? Why?

What does this tell us about the area of a parallelogram?